

Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 4 \\ -2 & 2 & 0 \end{pmatrix}.$$

Subtract 2 times row 1 from row 2. Subtract -2 times row 1 from row 3.

$$\rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & -2 \\ 0 & 6 & 6 \end{pmatrix}$$

Subtract 3 times row 2 from row 3.

$$\rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & -2 \\ 0 & 0 & 2 \end{pmatrix} = U$$

Place the multipliers in L .

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & 3 & 1 \end{pmatrix}$$

Note that we now have $A = LU$ with

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 4 \\ -2 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & -2 \\ 0 & 0 & 2 \end{pmatrix}.$$

Think in blocks. Then

$$\begin{pmatrix} A_k & * \\ * & * \end{pmatrix} = \begin{pmatrix} L_k & 0 \\ * & * \end{pmatrix} \begin{pmatrix} U_k & * \\ * & * \end{pmatrix}.$$

Then, clearly, $A_l = L_k U_k$ for $k = 1, 2$, and 3 . For example, $A_2 = L_2 U_2$; i.e.,

$$\begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}.$$

Clearly, $|L_2| = 1$, $|U_2| = 2$, and

$$|A_2| = |L_2| |U_2| = 1 \cdot 2 = 2.$$

With this and my previous note, you should be able to finish the exercise. :-)