Chris, this is a really good question, but I don't have a really definitive answer. However, I can share a few ideas.

In this case we are looking at the length of the error vector, and of course, to get the length we use the Euclidean definition of distance, which is the formula all algebra students learn for getting the distance between two points.

$$d((x_1, x_2), (y_2, y_2)) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2} \tag{1}$$

Of course, this is easily extended to higher dimensions.

$$d(x_1, \ldots, x_n), (y_1, \ldots, y_n)) = \sqrt{(x_1 - y_1)^2 + \cdots + (x_n - y_n)^2} \tag{2}$$

This is the so called *Euclidean Metric*, which measures the distance between two points. Note that it is based on the Theorem of Pythagoras. Using this metric, one can build something called a *metric space*. This is the normal metric space we use in our standard calculus courses.

However, as you have mentioned, there are other ways to measure the distance between two points. For example,

$$d((x_1, \ldots, x_n), (y_1, \ldots, y_n)) = |x_1 - y_1| + \cdots + |x_n - y_n| \tag{3}$$

Here's another example of a "metric" defined on the plane.

$$d(x_1, x_2), (y_1, y_2)) = max\{|x_1 - y_1|, |x_2 - y_2|\} \tag{4}$$

Each of these metrics is a definition of the "distance" between two points and each leads to some startling new geometries.

And, lest you think that these various definitions of distance are just the whims of bored mathematicians, let me provide a concrete example. Suppose that you have a city, where the streets are laid out in a grid, some streets running north and south, the remainder east and west. Now, imagine you are a taxi cab driver and you are parked on the corner of a city block at point A. Now, suppose that you have to travel to point B, diagonally opposite point A, around the corner of the block. Isn't it true, that as far as the taxicab driver is concerned, that the distance between points A and B is given by formula (3), and not the usual Euclidean definition of distance (formula (1))? That is, the distance between points A(x_1, x_2) and B(y_1, y_2) is

$$d((x_1, x_2), (y_1, y_2)) = |x_1 - y_1| + |x_2 - y_2|. \tag{5}$$

This leads to a new geometry called *Taxicab Geometry.* I have a book in my office describing this geometry more fully, a fun read.

We are using the familiar Euclidean metric in our class. Also, the error made in approximating the vector $\mathbf{b}$ with its projection $\mathbf{p}$ is the vector $\mathbf{e} = \mathbf{b} - \mathbf{p}$. The magnitude of the error is

$$\|e\| = \sqrt{e_1^2 + \cdots + e_n^2}. \tag{6}$$

Of course, minimizing this magnitude is accomplished by minimizing the squared error,

$$\|e\|^2 = e_1^2 + \cdots + e_n^2. \tag{7}$$

Hence, "Leasts Squares Error."

I hope this helps somewhat.