Both statements are correct. But Q need not be square.

Theorem 1 *Let $\{\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_k\}$ be an orthonormal set. The the matrix*

$$Q = \begin{pmatrix} \mathbf{q}_1 & \mathbf{q}_2 & \cdots & \mathbf{q}_k \end{pmatrix}$$

is orthogonal and $Q^T Q = I$.

Proof: Because $\{\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_k\}$ is an orthonormal set, we know that

$$\begin{cases} \mathbf{q}_i^T \mathbf{q}_j = 0, & \text{if } i \neq j, \\ \mathbf{q}_i^T \mathbf{q}_j = 1, & \text{if } i = j. \end{cases}$$

Thus,

$$\begin{aligned} Q^T Q &= \begin{pmatrix} \mathbf{q}_1^T \\ \mathbf{q}_2^T \\ \vdots \\ \mathbf{q}_n^T \end{pmatrix} \begin{pmatrix} \mathbf{q}_1 & \mathbf{q}_2 & \ldots & \mathbf{q}_n \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{q}_1^T \mathbf{q}_1 & \mathbf{q}_1^T \mathbf{q}_2 & \cdots & \mathbf{q}_1^T \mathbf{q}_n \\ \mathbf{q}_2^T \mathbf{q}_1 & \mathbf{q}_2^T \mathbf{q}_2 & \cdots & \mathbf{q}_2^T \mathbf{q}_n \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{q}_n^T \mathbf{q}_1 & \mathbf{q}_n^T \mathbf{q}_2 & \cdots & \mathbf{q}_n^T \mathbf{q}_n \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \\ &= I. \end{aligned}$$

The converse is also true.

Theorem 2 *If $Q^T Q = I$, then the columns of Q form an orthonormal set.*

Proof: Suppose that

$$Q^T Q = I.$$

This is the same as saying

$$\begin{pmatrix} \mathbf{q}_1^T \\ \mathbf{q}_2^T \\ \vdots \\ \mathbf{q}_n^T \end{pmatrix} \begin{pmatrix} \mathbf{q}_1 & \mathbf{q}_2 & \ldots & \mathbf{q}_n \end{pmatrix} = I,$$

or, equivalently,

$$\begin{pmatrix} \mathbf{q}_1^T \mathbf{q}_1 & \mathbf{q}_1^T \mathbf{q}_2 & \cdots & \mathbf{q}_1^T \mathbf{q}_n \\ \mathbf{q}_2^T \mathbf{q}_1 & \mathbf{q}_2^T \mathbf{q}_2 & \cdots & \mathbf{q}_2^T \mathbf{q}_n \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{q}_n^T \mathbf{q}_1 & \mathbf{q}_n^T \mathbf{q}_2 & \cdots & \mathbf{q}_n^T \mathbf{q}_n \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}.$$

Clearly, when you compare these matrices,

$$\begin{cases} \mathbf{q}_i^T \mathbf{q}_j = 0, & \text{if } i \neq j, \\ \mathbf{q}_i^T \mathbf{q}_j = 1, & \text{if } i = j, \end{cases}$$

and $\{\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_k\}$ is an orthonormal set.