

Strang

Exercise 19, Page 12

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Consider

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Note that the second column is a multiple of the first. Hence, if we denote the columns of matrix A by \mathbf{a}_1 and \mathbf{a}_2 , then note that

$$\mathbf{a}_2 = 2\mathbf{a}_1.$$

Hence, any linear combination of \mathbf{a}_1 and \mathbf{a}_2 will simply be a multiple of \mathbf{a}_1 . This is easily shown.

$$\begin{aligned} \alpha\mathbf{a}_1 + \beta\mathbf{a}_2 &= \alpha\mathbf{a}_1 + \beta(2\mathbf{a}_1) \\ &= (\alpha + 2\beta)\mathbf{a}_1. \end{aligned}$$

Now, the column space of A is the set of all linear combinations of the columns of A . But we've already shown that any linear combination of the columns of A is simply a multiple of the first column of A . Therefore, the column space of A will be a line through the origin in the direction of the first column of A .

This is easily shown in Matlab. First, enter the first column of matrix A .

```
>> a=[1;0;0]
```

```
a =
```

```
1
0
0
```

Now, let's get a 100 random multiples of this column, then plot them. They should lie on a line through the origin in the direction of $\mathbf{a} = (1, 0, 0)^T$. This is also easily done. First, create a row vector of size 100 with random numbers between -5 and 5 .

```
>> b=10*rand(1,100)-5;
```

Now, if you are thinking in terms of block multiplication, you know what \mathbf{ab} will look like. For example, suppose that

$$\mathbf{bb} = [1 \quad 2 \quad -3 \quad 4 \quad -2].$$

Then,

$$\mathbf{ab} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [1 \quad 2 \quad -3 \quad 4 \quad -2] = \begin{bmatrix} 1 & 2 & -3 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Isn't block multiplication wonderful?

Now, \mathbf{ab} will contain a hundred multiples of column \mathbf{a}_1 in its columns.

```
>> c=a*b;
```

Now, let's plot each column of \mathbf{c} as a point in three space. We will use Matlab's `line` command for this purpose and some of Matlab's handle graphics to choose the line and marker style.

```
>> close all
>> for k=1:100
line(c(1,k),c(2,k),c(3,k),'LineStyle','none','Marker','*')
end
```

Now, we'll rotate the view, add a box for some depth, then add a grid.

```
>> view(135,50)
>> box on
>> grid on
```

Finally, add some axis labels.

```
>> xlabel('x-axis'),ylabel('y-axis')
```

All of this gives the image in Figure 1.

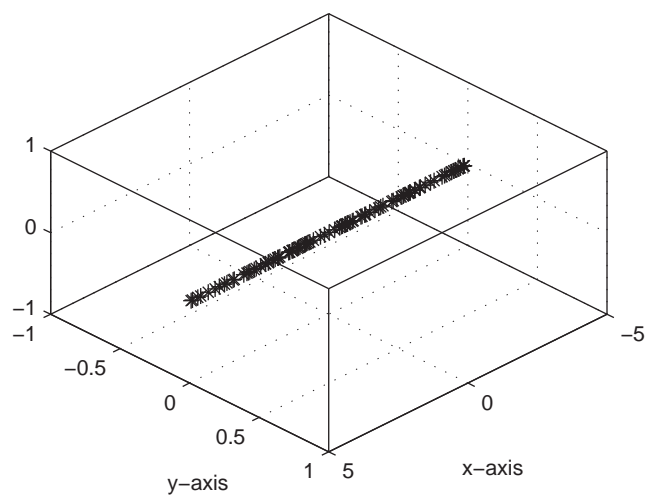


Figure 1: The column space is a line through the origin.