

On page 131, we have the augmented matrix

$$(R \ \mathbf{d}) = \begin{pmatrix} 1 & 3 & 0 & 2 & 1 \\ 0 & 0 & 1 & 4 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

This augmented matrix represents the system of equations

$$\begin{aligned} x_1 + 3x_2 + 2x_4 &= 1 \\ x_3 + 4x_4 &= 6 \end{aligned}$$

Solving these for the pivot variables,

$$\begin{aligned} x_1 &= 1 - 3x_2 - 2x_4 \\ x_3 &= 6 - 4x_4. \end{aligned}$$

Note that  $x_2$  and  $x_4$  are free. Let  $x_2 = x_4 = 0$ . Then,

$$\begin{aligned} x_1 &= 1 \\ x_3 &= 6. \end{aligned}$$

Thus, a solution is

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 6 \\ 0 \end{pmatrix}.$$

Now, Strang says, and I agree, why do we have to write the equations down, solve them for the pivot variables, and substitute 0 for the free variables to get a particular solution for the system? Can't we just look at the matrix and write down this particular solution?

You betcha! Just grab the numbers from  $\mathbf{d}$  for the pivot variables, then put zeros into the free variable locations, and you have a particular solution.

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 6 \\ 0 \end{pmatrix}.$$

Easy, peasy!