Strang, Page 132, Exercise #19

David Arnold

October 15, 2006

When working with proof, you have to learn how to explain yourself with sound mathematical writing. For example, consider exercise #19 on page 132 of Strang:

> "If L is invertible and $A = LU$, show that $N(A) = N(U)$."

Proof. Assume that L is invertible and $A = LU$.[1]. To show that The nullspace of A and U are equal (i.e, $N(A) = N(U)$), we have to show two things:

1. $N((U) \subset N(A)$, and
2. $N(A) \subset N(U)$.

Let's first show that $N(U) \subset N(A)$. To do this, we select an arbitrary element in $N(U)$ and show that it is also in $N(A)$.

So, let $\mathbf{x} \in N(U)$. This implies that

$$U\mathbf{x} = \mathbf{0}.$$

Multiply both sides of this equation by L.

$$LU\mathbf{x} = L\mathbf{0}$$

However, $A = LU$ and $L\mathbf{0} = \mathbf{0}$. Substituting these in the last equation give us

$$A\mathbf{x} = \mathbf{0}.$$

Hence, $\mathbf{x} \in N(A)$. Therefore, each object in $N(U)$ is also in the space $N(A)$. Therefore, $N(U) \subset N(A)$.

I will leave the proof of the second part (that $N(A) \subset N(U)$) for you to do.

Finally, because $N(U) \subset N(A)$ and $N(A) \subset N(U)$, we know that the two nullspaces are identical, i.e., $N(A) = N(U)$.

This is the kind of writing and effort that I am looking for on proofs. I know it's hard and it's something that you are not used to doing, but it is also something that you must strive for.

[1] You always start a proof of an if-then statement by assuming the "if part" (the hypothesis)