

Think about the dimensions. For example, suppose that

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}.$$

Note that there are 2 rows and 3 columns. In order that the matrix-vector multiplication $A\mathbf{x}$ be well defined, vector \mathbf{x} must have 3 rows and 1 column, as in

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}.$$

If you think in terms of the “column picture,” this is clear.

$$\begin{aligned} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} &= 1 \begin{pmatrix} 1 \\ 4 \end{pmatrix} - 1 \begin{pmatrix} 2 \\ 5 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ 6 \end{pmatrix} \\ &= \begin{pmatrix} 4 \\ 11 \end{pmatrix}. \end{aligned}$$

Therefore, matrix A , having 2 rows and 3 columns, multiplies a vector with 3 components to produce a vector with 2 components.

The generalization in exercise 13 on page 31 should now be clear.