How about this trivial answer:

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Certainly, regardless of the matrix A, we have

$$BA = 4B.$$

But the real question is, can I find another matrix B such that

$$BA = 4B$$

for any matrix A? That's a harder question. So, let

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}.$$

Now, Lee, you had a pretty good idea going. That is, if

$$BA = 4B$$

for all A, then

$$BA - 4B = 0$$

for all A. You tried to factor, but

$$B(A - 4) = 0$$

cannot be correct, because $A - 4$ makes no sense. You cannot subtract a scalar from a matrix. However, consider what happens when you insert

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Then, one can argue that

$$\begin{aligned} BA &= 4B \\ BA &= 4BI \\ BA - 4BI &= 0 \\ B(A - 4I) &= 0. \end{aligned}$$

Now, this needs to be true for all A, so it will be true for

$$A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}.$$

However,

$$A - 4I = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} - 4 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I.$$

Thus,

$$B(A - 4I) = 0$$

becomes

$$BI = 0$$

or

$$B = 0.$$

Thus, there is no nonzero matrix that will do the job required.