Another cute problem! First, note in part (b) that A is singular. If A were nonsingular (invertible), then by part (a), B would have to equal C. Now,

$$\begin{aligned} AB &= AC \\ AB - AC &= 0 \\ A(B-C) &= 0. \end{aligned}$$

If $B - C = 0$, then $B = C$. We don't want this, so we want to find B and C such that $B - C \neq 0$. Let's let U temporarily represent $B - C$.

$$U = B - C$$

Because U is arbitrary, it could be anything. Let

$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Now, $A(B - C) = 0$, so

$$\begin{aligned} AU &= 0 \\ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & 2 \end{pmatrix} &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} a+c & b+d \\ a+c & b+d \end{pmatrix} &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \end{aligned}$$

Thus,

$$\begin{aligned} a + c &= 0 \\ b + d &= 0, \end{aligned}$$

and it must be the case that $c = -a$ and $d = -b$. Therefore, U must have the following form.

$$U = \begin{pmatrix} a & b \\ -a & -d \end{pmatrix}$$

Now, suppose that B is any matrix at all, as in

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}.$$

Because $U = B - C$, it must be that

$$\begin{aligned} C &= B - U \\ C &= \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} - \begin{pmatrix} a & b \\ -a & -b \end{pmatrix} \\ C &= \begin{pmatrix} b_{11} - a & b_{12} - b \\ b_{21} + a & b_{22} + b \end{pmatrix}. \end{aligned}$$

It is easily checked that $AB = AC$ for these arbitrary choices of B and C.

Moreover, you can craft *any* particular example with these templates. For example, if

$$B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad \text{and} \qquad U = \begin{pmatrix} 5 & 6 \\ -5 & -6 \end{pmatrix},$$

then

$$C = B - U = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} 5 & 6 \\ -5 & -6 \end{pmatrix} = \begin{pmatrix} -4 & -4 \\ 8 & 10 \end{pmatrix}.$$

It is easily checked that $AB = AC$. The difference in my approach is I am finding all possible examples with $AB = AC$, but $B \neq C$, not just one particular example.