

Suppose that you reduce A to upper triangular U with this sequence of elementary matrices.

$$E_{32}E_{31}E_{21}A = U$$

Now, multiply both sides of this equation on the left by $(E_{32}E_{31}E_{21})^{-1}$. This gives

$$\begin{aligned}(E_{32}E_{31}E_{21})^{-1}(E_{32}E_{31}E_{21})A &= (E_{32}E_{31}E_{21})^{-1}U \\ A &= (E_{32}E_{31}E_{21})^{-1}U.\end{aligned}$$

The inverse of a product is the product of the inverses, but in reverse order.

$$A = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}U$$

Let

$$L = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}.$$

Then, $A = LU$.