You asked, "Is that the only way to multiply." Indeed, the answer is no. Depending on how you block out the matrices, there are many flavors of matrix multiplication. In class today, I wanted to make the point that if I multiply a row vector time a matrix, I get a row vector. For example,

$$\begin{pmatrix}1 & 2 & 2\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{pmatrix} = \begin{pmatrix}30 & 36 & 42\end{pmatrix}.$$

I justified this product as follows. First, note that the row has dimension 1×3, while the matrix has dimension 3×3. Hence, the inner dimensions match, so the multiplication is possible. Further, the outer dimensions give the dimension of the product, 1×3.

Then I wanted to show the procedure for multiplying a row vector times a matrix, that is, one takes a linear combination of the rows of the matrix, using the entries in the row vector as weights. This is justified by cutting the matrices into blocks as follows.

$$\begin{aligned}\begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{pmatrix} &= \left(\begin{array}{c|c|c}1 & 2 & 3\end{array}\right)\left(\begin{array}{c}1 & 2 & 3\\ \hline 4 & 5 & 6\\ \hline 7 & 8 & 9\end{array}\right)\\ &= \begin{pmatrix}A_1 & A_2 & A_3\end{pmatrix}\begin{pmatrix}B_1\\ B_2\\ B_3\end{pmatrix}\\ &= A_1B_1 + A_2B_2 + A_3B_3\\ &= 1\begin{pmatrix}1 & 2 & 3\end{pmatrix} + 2\begin{pmatrix}4 & 5 & 6\end{pmatrix} + 3\begin{pmatrix}7 & 8 & 9\end{pmatrix}\\ &= \begin{pmatrix}1 & 2 & 3\end{pmatrix} + \begin{pmatrix}8 & 10 & 12\end{pmatrix} + \begin{pmatrix}21 & 24 & 27\end{pmatrix}\\ &= \begin{pmatrix}30 & 36 & 42\end{pmatrix}.\end{aligned}$$

However, this is not the only way we can multiply this row vector times our matrix, though I think this is the most efficient way to think: multiply on the left, row operations; multiply on the right, column operations. We could have partitioned and multiplied as follows.

$$\begin{aligned}\begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{pmatrix} &= \begin{pmatrix}1 & 2 & 3\end{pmatrix}\left(\begin{array}{c|c|c}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{array}\right)\\ &= A\begin{pmatrix}B_1 & B_2 & B_3\end{pmatrix}\\ &= \begin{pmatrix}AB_1 & AB_2 & AB_3\end{pmatrix}\\ &= \left(\begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}1\\ 4\\ 7\end{pmatrix} \quad \begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}2\\ 5\\ 8\end{pmatrix} \quad \begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}3\\ 6\\ 9\end{pmatrix}\right)\\ &= \begin{pmatrix}30 & 36 & 42\end{pmatrix}.\end{aligned}$$