We are looking for a nonzero solutions of

$$A^2 = 0.$$

Of course, this means that we are looking for a nonzero solution of

$$AA = 0.$$

If we can find such an A that is nonsingular, then it is invertible and

$$\begin{aligned} A^{-1}(AA) &= A^{-1}0 \\ (A^{-1}A)A &= 0 \\ IA &= 0 \\ A &= 0 \end{aligned}$$

Of course, this is a contradiction, because the zero matrix is not invertible (it's singular).

Thus, in the 2×2 case, if A is a nonzero solution of $AA = 0$, then A is singular. Thus, the second row must be a multiple of the first and the matrix must look like this:

$$A = \begin{pmatrix} a & b \\ ka & kb \end{pmatrix}$$

Now, $AA = 0$, so

$$\begin{aligned} \begin{pmatrix} a & b \\ ka & kb \end{pmatrix} \begin{pmatrix} a & b \\ ka & kb \end{pmatrix} &= \begin{pmatrix} a^2 + kab & ab + kb^2 \\ ka^2 + k^2ab & kab + k^2b^2 \end{pmatrix} \\ &= \begin{pmatrix} a(a + kb) & b(a + kb) \\ ka(a + kb) & kb(a + kb) \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \end{aligned}$$

Let's assume for the moment that $a \neq 0$ and $b \neq 0$ (are their other cases?). Then we must have that

$$\begin{aligned} a + kb &= 0 \\ a &= -kb. \end{aligned}$$

Then, with $a = -kb$, we have

$$A = \begin{pmatrix} a & b \\ ka & kb \end{pmatrix} = \begin{pmatrix} -kb & b \\ -k^2b & kb \end{pmatrix}.$$

For example, with $k = 2$ and $b = 1$, we have

$$A = \begin{pmatrix} -kb & b \\ -k^2b & kb \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -4 & 2 \end{pmatrix}.$$

Checking,

$$\begin{pmatrix} -2 & 1 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

You can get a horde of answers by varying k and b. Now what about other cases?